AERODYNAMIC TORQUE ON A SPINNING SPHERICAL SATELLITE WITH APPLICATION TO MEASUREMENT OF ACCOMMODATION COEFFICIENTS

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LIST OF IMPORTANT SYMBOLS

Symbol	
I	moment of inertia of satellite
i	orbit inclination
L	torque about satellite center of mass
n	number of complete orbits
$\bar{n}$	unit vector normal to satellite surface
R	radius of satellite
R_e	radius of earth
R_o	radius of orbit measured from center of earth
T_w	temperature of satellite surface
V	satellite orbital velocity
α	angle between negative gas flow velocity vector and reference line in orbital plane
α_d	accommodation coefficient
ϵ	angle between satellite spin axis and the earth's north pole
θ_s	angle between satellite spin axis and the normal to orbit
ρ	air density
ρ_s	material density of satellite
ϕ	angle between satellite "line of nodes" and reference line in earth equatorial plane
ϕ_s	angle between satellite "line of nodes" and line of nodes of orbit
$\dot{\Phi}$	orbital regression rate
ω_o	orbital angular velocity

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I. INTRODUCTION

A spinning spherical satellite has been proposed by the Coordinated Science Laboratory to measure a general relativity effect which is predicted to cause the spin axis of the satellite to precess 5 to 7 sec of arc per year.[1] Measurement of the precession is realized by a unique method or readout which utilizes terrestrial sightings of sunlight reflected from mirrors on the satellite's surface. The measurement of such a small precession rate required an investigation of other torques producing effects which could possibly prevent the isolation of the relativity effect. In this respect, the analysis to determine the aerodynamic torque was initiated.

The aerodynamic torque on a spinning spherical satellite has also been studied by R. D. Palamara[2] and Nan Tum Po.[3] R. D. Palamara analyzed not only aerodynamic torque but also numerous other effects which cause torque on an unprotected spinning satellite. His study of the aerodynamic torque is, however, too restricted in that the satellite spin axis is taken as being in the plane of the orbit. Nan Tum Po obtained a more general solution which he used to find the spin rate slowdown of a satellite having high surface area to mass ratio. The Coordinated Science Laboratory is, however, proposing to measure the precession rate of a solid spherical satellite for which the qualitative results of Nan Tum Po do not apply.

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[1] Coordinated Science Laboratory, University of Illinois, Urbana, "Quarterly Progress Report" for Dec., 1964, Jan. and Feb., 1965, pp. 1-12.

[2] R. D. Palamara, Synthesis of a General Relativity Experiment (Thesis), June 30, 1964, pp. 53-74.

[3] Nan Tum Po, "On the Rotation Motion of a Spherical Satellite Under the Action of Retarding Aerodynamical Moments," NASA TTF-9630, January 22, 1965.

There was then an expressed need for a general analysis of the aerodynamic effect which could be applied to the proposed satellite.

The study consists of first obtaining the general analytical expression for aerodynamic torque which is found to depend upon the orientation of the satellite and the accommodation coefficient of the surface. Consideration is then given to the effect of nonuniform distribution of accommodation coefficient and orbital regression effects which cause a change in satellite orientation with time. When these results are applied to the CSL satellite, two conclusions are evident. First, a reasonable adjustment in the initial orbital parameters and satellite orientation can be made so as to reduce the aerodynamic effect to a value acceptable for the relativity experiment. Second, aerodynamic effect can also be simplified to make possible a direct measurement of the accommodation coefficient of the satellite surface. The feasibility of performing a satellite experiment to measure the accommodation coefficient by the technique proposed by the Coordinated Science Laboratory is investigated.

The obvious advantage the satellite method has over earthbound-laboratory methods of measuring the accommodation coefficient is that the measurements are made under actual orbital conditions. The extreme difficulty involved in attempting to simulate orbital conditions led William J. Evans to say that "with improved laboratory techniques enabling better simulation of orbital conditions, it is entirely possible that all prior data (on accommodation coefficients) will have been discredited."[4]

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[4]William J. Evans, "Aerodynamic and Radiation Disturbance Torques on Satellites Having Complex Geometry," Torques and Attitude Sensing in Earth Satellites, ed. S. Fred Siner, 1964, p. 85.

The importance and difficulty of simulation of orbital conditions can be derived from the concluding remarks of Harold Y. Wachman.

> Accommodation coefficient values that can be confidently applied to a solution of satellite drag and heat transfer problems must be obtained from experiments with gas molecules whose velocities match those of satellites. To the author's knowledge, the problem of generating in the laboratory a nearly mono-energetic beam of molecules in the range 1 to 10 eV with adequate flux has not yet been solved.[5]

Daniel McKeowan sights the problems of laboratory techniques and the advantages of using satellite methods.

> Using the latest advances in vacuum technology, it is possible to obtain pressures better than 10^{-9} Torr. The moment the beam is turned on, though, the vacuum is degraded to pressures of 10^{-6} Torr due to gas flow. At this pressure the test surface is usually covered with many layers of absorbed gas which shield it from direct beam bombardment. The results of a laboratory experiment are difficult to interpret in contrast to a clean surface such as the skin of a satellite.[6]

Therefore, a satellite experiment eliminates the laboratory problems by embodying the actual orbital conditions. This advantage coupled with the accuracy and unique simplicity of the CSL satellite provides a superior experimental method for the study of accommodation coefficients.

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[5] Harold Y. Wachman, "The Thermal Accommodation Coefficient: A Critical Survey," ARS Journal 32, January, 1962, p. 11.

[6] Daniel McKeowan, "Surface Erosion in Space," Rarefied Gas Dynamics, Ed. J. A. Laurman, Supplement 2, Vol. I, 1963, p.

II. ANALYSIS OF AERODYNAMIC TORQUE ON A SPINNING SPHERICAL SATELLITE

The satellite analyzed is the type proposed by the Coordinated Science Laboratory to measure a general relativity effect. The basic design and readout system of the CSL satellite consists of utilizing terrestrial sightings of sunlight reflected from mirrors on the satellite's nearly spherical surface.[7,8] Proposed design parameters consider the satellite to be about one foot in diameter, composed of solid glass and spinning at about 250 rev. per sec. These parameters will be used in the analysis to present sample results. Although the satellite is polyhedral in design, it is assumed to be spherical for purposes of analysis.

The satellite orbit must be restricted to altitudes of less than 1000 miles to produce a measurable relativity effect and also to provide sufficient brightness of reflected sunlight for data collection. This restriction to low orbits allows the assumption to be made that random or thermal motion of gas molecules can be neglected in comparison to the relative velocity of the satellite;[9] therefore, the incident velocity of gas molecules can be taken to be equal to the orbital velocity of the satellite. The assumption of free molecular flow is valid above 100 miles altitude and therefore can be used in the altitudes of interest for this analysis.[10]

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[7] CSL "Quarterly Progress Report," op. cit.

[8] D. H. Cooper, "Passive Polyhedral Gyro Satellite," Coordinated Science Laboratory Report I-128, February 16, 1963.

[9] Wachman, op. cit. p. 2.

[10] Evans, op. cit., p. 83.

Since the flow is free molecular, the aerodynamic forces may be calculated by considering the incident and reflected molecules separately. The reflection of molecules is, however, determined by the accommodation coefficient of the surface. In this analysis the classical Maxwell accommodation coefficient is used in which the accommodation coefficient (α_d) equals the percentage of impinging molecules that are diffusely reflected from the surface after being accommodated to the surface. The remaining percentage, $(1 - \alpha_d)$, are specularly reflected with no energy accommodation.

The torque caused by gas surface interactions is analyzed by viewing the specularly and diffusely reflected molecules separately. The resultant force due to a specular reflection is normal to the surface; therefore, for a spherical surface, no torque about the center of mass results from specularly reflected molecules. For the diffusely reflected molecules, the torque is evaluated in the following way. First, the force per unit area due to the impingement of the molecule depends on the angle of the surface to the flow and in general causes a torque about the center of mass. Second, we take into consideration the dynamical effect of the reflection. After accommodation to the surface, the molecule is diffusely reflected with a velocity component normal to the surface. This component results in only a normal force as required by the definition of diffuse reflection. Since the molecules have been accommodated to the surface, each diffusely reflected molecule has also a component of velocity tangent to the surface and equal to the angular velocity of the surface at the point of reflection. It is these mechanisms which produces the precessional torque that causes the directional movement of the spin axis.

The coordinate systems to be used in the analysis are shown in Figs. 1 and 2. The X, Y, Z axis is the inertial set with Z toward Polaris and X along the vernal equinox. The x, y, z coordinate system is attached to the orbit with the z axis normal to the orbital plane and the x axis as the ascending node of the orbit. The x_s, y_s, z_s set is attached to the gyro and the x_s axis can be thought of as the line formed where the plane normal to the spin axis intersects the orbital plane. i, j, k, and i_s, j_s, k_s are unit vectors along the x, y, z and x_s, y_s, z_s axes respectively. Also shown in the figures is the gas flow velocity vector which is always in the x,y plane and can be thought to rotate about the center of the satellite at the orbital angular velocity. The velocity vector is tangent to the orbital path at all times. Fig. 3 illustrates the spherical coordinate system (η,ξ,R) used in the integration over the surface of the sphere.

The mass flux of gas molecules impinging upon an element of surface dA is

$$\rho\overline{V} \cdot \overline{n}\, dA \tag{1}$$

where ρ is the atmospheric density at the orbital altitude, $\overline{V}$ is the velocity vector equal to the orbital velocity, and $\overline{n}$ is the normal to the surface area dA.

Since only the diffusely reflected molecules enter in the torque analysis, we may ignore the specularly reflected ones. The impinging molecules exert a torque

$$-(\overline{R}\times\overline{V})\, \alpha_d\, (\rho\overline{V} \cdot \overline{n}\, dA)\ . \tag{2}$$

The reflecting molecules exert a torque

$$\overline{R}\times(\overline{\Omega}\times\overline{R})\, \alpha_d\, (\rho\overline{V} \cdot \overline{n}\, dA) \tag{3}$$

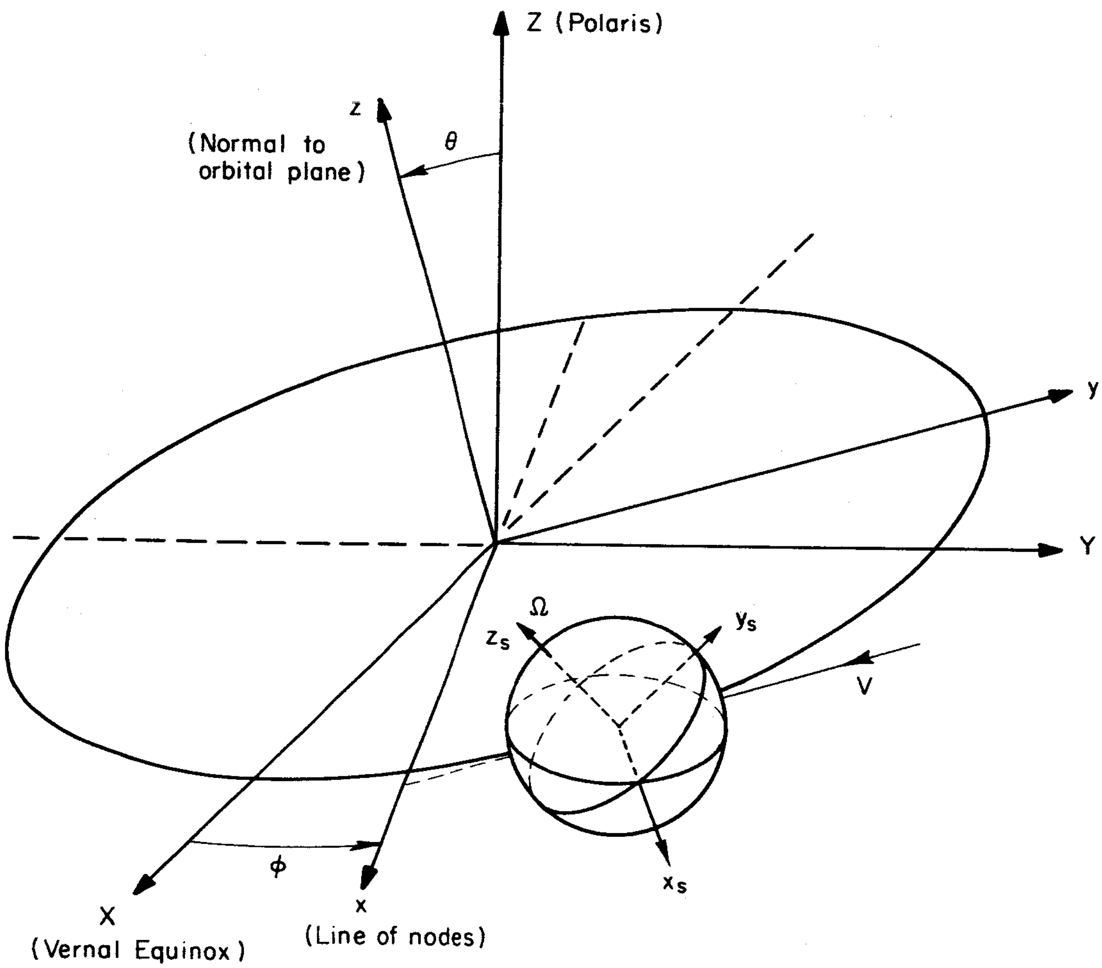

Figure 1. Arrangement of coordinate systems.

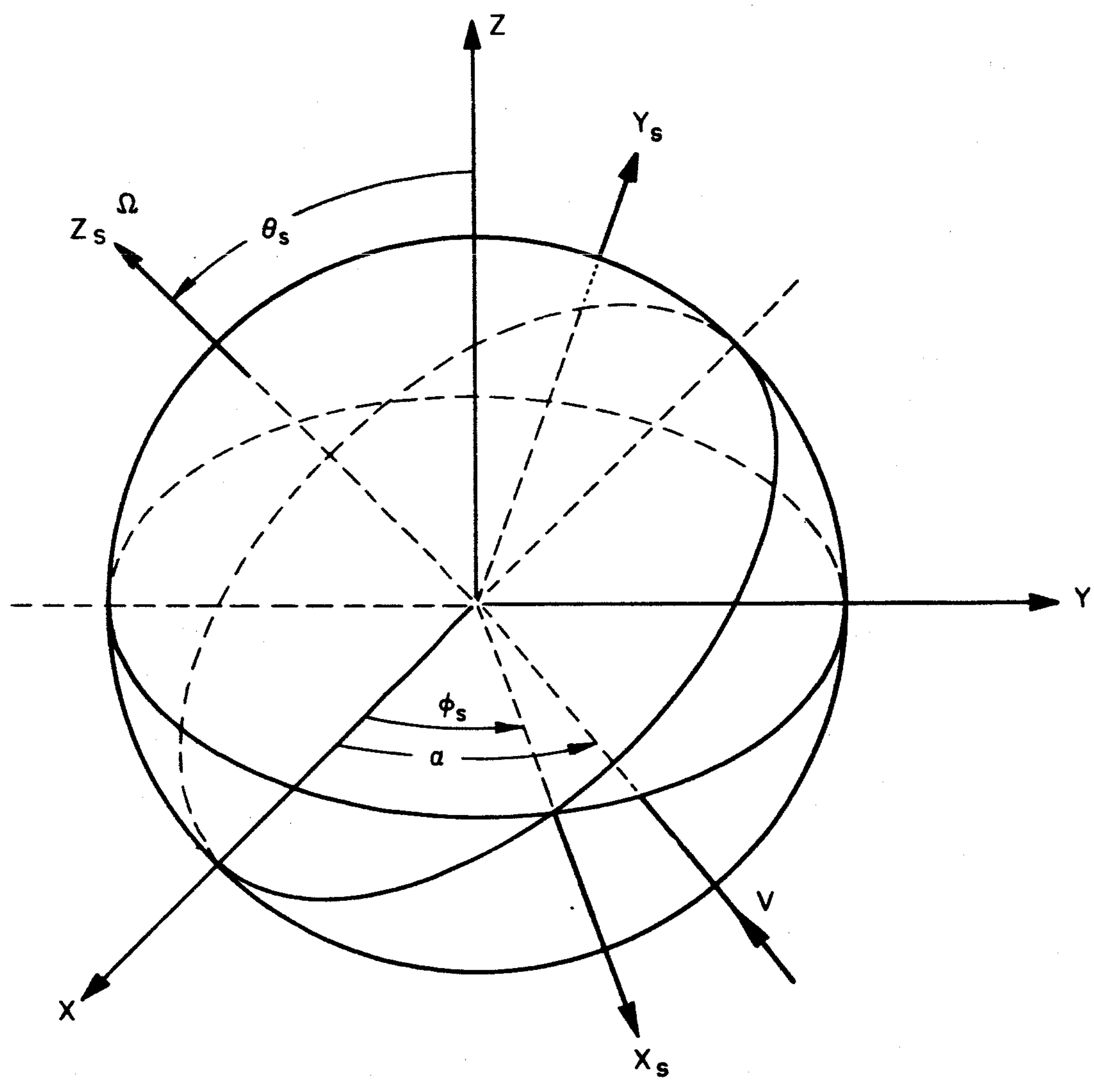

Figure 2. Coordinate systems transported to a common origin.

where $\overline{\Omega}$ is the spin vector of the gyro and $\overline{R}$ is the radius vector to the surface from the center of mass. The elemental torque $d\overline{L}$ produced by the complete interaction is then the sum of the above expressions, i.e.,

$$d\overline{L} = - \alpha_d \left[\overline{R}\times(\overline{V} - \overline{\Omega}\times\overline{R}) \; (\rho\overline{V} \cdot n \; dA)\right]. \tag{4}$$

Referring now to Figs. 2 and 3 we define the following vectors and terms:

$$\begin{aligned}
\overline{\Omega} &= \Omega k_s = \Omega \sin\theta_s \sin\phi_s \; i - \Omega \sin\theta_s \cos\phi_s \; j + \Omega \cos\theta_s \; k \\
\overline{V} &= -V \cos\alpha i - V \sin\alpha \; j \\
dA &= R^2 \sin\eta \; d\eta \; d\xi \\
\overline{R} &= R \sin\eta \cos\xi \; i + R \sin\eta \sin\xi \; j + R \cos\eta \; k \\
R &= \text{radius of sphere} \\
\overline{n} &= \sin\eta \cos\xi \; i + \sin\eta \sin\xi \; j + \cos\eta \; k.
\end{aligned} \tag{5}$$

We substitute these vectors into the expression for $d\overline{L}$ and integrate over only the surface area in the velocity stream. This is accomplished by first integrating over η from 0 to π. We then integrate over ξ from $\alpha - \frac{\pi}{2}$ to $\alpha + \frac{\pi}{2}$ because α defines the direction of the velocity vector $\overline{V}$. After integration the instantaneous torque reduces to a function of the angles α, θ_x, and ϕ_s.

$$\begin{aligned}
\overline{L} = &- L_o \left[\sin\theta_s \sin\phi_s(\sin^2\alpha + 2) + \sin\theta_s \cos\phi_s(\sin\alpha\cos\alpha)\right]i \\
&+ L_o \left[\sin\theta_s \sin\phi_s(\sin\alpha\cos\alpha) + \sin\theta_s \cos\phi_s(\cos^2\alpha + 2)\right]j \\
&+ 3L_o \left[\cos\theta_s\right]k
\end{aligned} \tag{6}$$

where

$$L_o = \alpha_d \frac{\rho V \pi R^4}{4} \Omega . \tag{7}$$

If we now consider the orbit fixed in inertial space, the problem is similar to the classic top or gyro problem where the torque is defined with respect to a nonmoving frame. To find the resulting motion of the spin axis under the action of the above torque, we must first find its

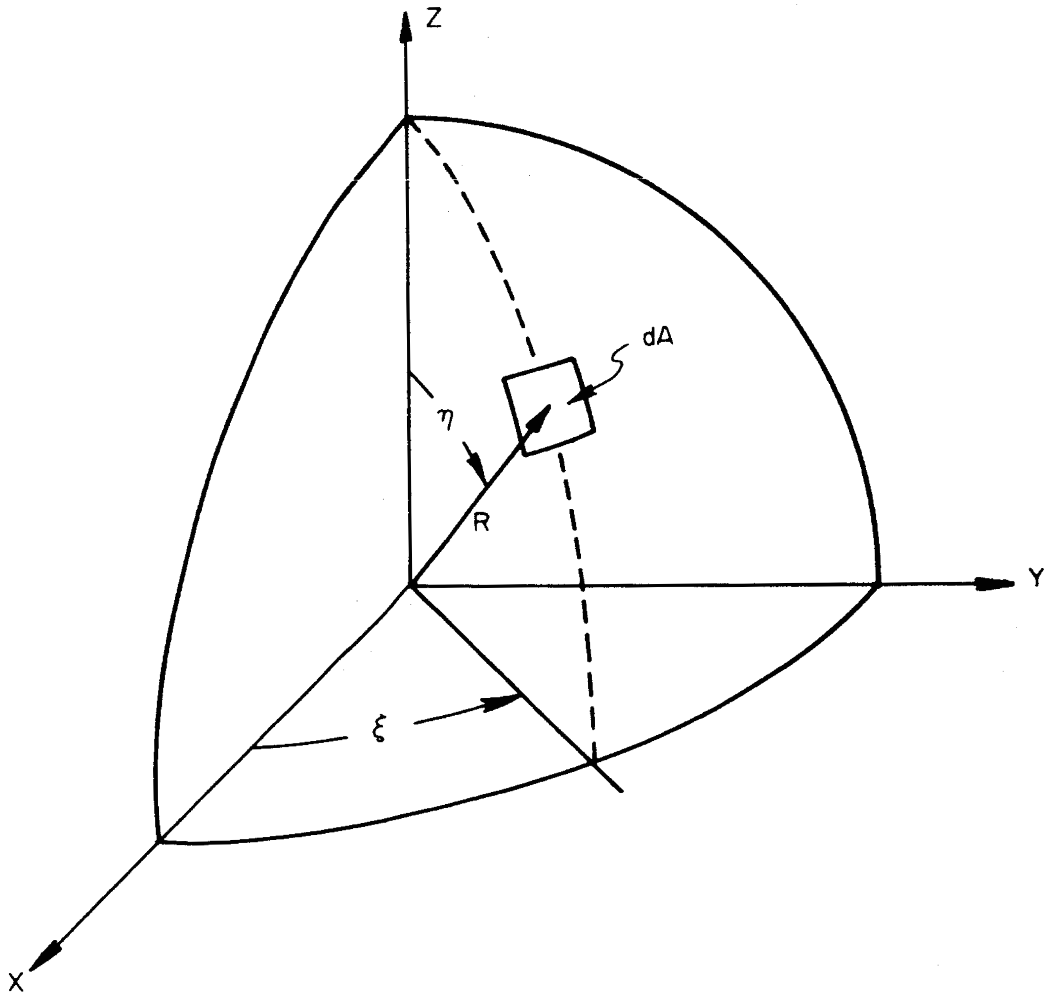

Figure 3. Coordinate system used for surface integration.

components in the x_s, y_s, z_s system. The torque referred to in this system, $\overline{L}_s$, becomes

$$\overline{L}_s = L_o[\sin\theta_s \sin\phi_s \cos\phi_s(\cos^2\alpha - \sin^2\alpha) + \sin\theta_s \sin\alpha \cos\alpha$$
$$(\sin^2\phi_s - \cos^2\phi_s)]i_s$$
$$+ L_o[\sin\theta_s\cos\theta_s\cos^2\phi_s(\cos^2\alpha+2) + \sin\theta_s \cos\theta_s\sin^2\phi_s(\sin^2\alpha+2)$$
$$+ 2 \sin\theta_s \cos\theta_s \sin\phi_s\cos\phi_s \sin\alpha \cos\alpha - 3 \cos\theta_s \sin\theta_s]j_s \tag{8}$$
$$+ L_o[-2 \sin^2\theta_s \sin\phi_s \cos\phi_s \sin\alpha \cos\alpha - \sin^2\theta_s \cos^2\phi_s(\cos^2\alpha+2)$$
$$- \sin^2\theta_s \sin^2\phi_s(\sin^2\alpha+2) - 3 \cos^2\theta_s]k_s \quad .$$

Since the spin rate of the gyro will be much higher than the angular movements of the spin axis, the Euler equations can be simplified by neglecting terms of small magnitude. The precession of the spin axis is then defined by the following:

$$\dot{\theta}_s = - \frac{L_{y_s}}{I\Omega} \tag{9}$$

$$\dot{\phi}_s \sin\theta_s = \frac{L_{x_s}}{I\Omega} \tag{10}$$

where L_{y_s} and L_{x_s} are the y_s and x_s components of torque, respectively, and I is the moment of inertia about the spin axis. The angular displacement for any time t becomes

$$\Delta\theta_s = \int_o^t - \frac{L_{y_s}}{I\Omega}\, dt \tag{11}$$

and

$$\Delta\phi_s = \int_o^t \frac{L_{y_s}}{\sin\theta_s\, I\Omega}\, dt \quad . \tag{12}$$

Now, for a circular orbit $\alpha = \omega_o t$ where ω_o is the constant orbital angular velocity. Therefore,

$$d\alpha = d\omega_o t = \omega_o dt$$

$$dt = \frac{d\alpha}{\omega_o} .$$

When $t = 0$ say $\alpha = 0$ and when $t = T$ = time over which measurements are made, $\alpha = 2n\pi$, where n is the number of completed orbits in the time T. Therefore,

$$\Delta\theta_s = \int_o^{2n\pi} - \frac{L_{y_s}}{I\Omega} \frac{d\alpha}{\omega_o}$$

$$\Delta\phi_s = \int_o^{2n\pi} \frac{L_{x_s}}{\sin\theta_s \; I\Omega} \frac{d\alpha}{\omega_o} .$$

From equation (8), the functions of α are seen to be periodic within the range 0 to 2π. The integrals from 0 to $2n\pi$ can be expressed as n times the integral from 0 to 2π. Also contained in the expressions for $\dot{\theta}_s$ and $\dot{\phi}_s$ are terms containing the trigonometric functions of θ_s and ϕ_s. Since the change in θ_s and ϕ_s is small over the time of interest, the trigonometric functions of these angles can be considered constant during integration, and we have

$$\Delta\theta_s = \frac{n}{I\Omega\omega_o} \int_o^{2\pi} - L_{y_s} \, d\alpha \tag{13}$$

$$\Delta\phi_s = \frac{n}{\sin\theta_s I\Omega\omega_o} \int_o^{2\pi} L_{x_s} \, d\alpha . \tag{14}$$

Now, performing the integration

$$\int_o^{2\pi} L_{x_s} \, d\alpha = 0 \tag{15}$$

while

$$\int_o^{2\pi} - L_{y_s} \, d\alpha = \pi L_o \sin\theta_s \cos\theta_s . \tag{16}$$

Therefore,

$$\Delta\theta_s = \frac{n\pi L_o}{I\Omega\omega_o} \sin\theta_s \cos\theta_s \tag{17}$$

and the movement of the spin axis is a precession causing the spin axis to move into the plane of the orbit if initially out of the plane. However, no precession exists if the spin axis is initially perpendicular to the orbital plane ($\theta_s = 0$) or if the spin axis is in the orbital plane ($\theta_s = \pi/2$). The maximum value of $\Delta\theta$ occurs when $\theta_s = \pi/4$. Substituting into equation (17) the expression for L_o equation (7) and note the spin rate Ω is not contained in the result, we obtain

$$\Delta\theta_s = \frac{m\, \alpha_d \rho V\, \pi^2 R^4}{8I\, \omega_o} \sin 2\theta_s \ . \tag{18}$$

Now, for a solid sphere made of material density ρ_s, the moment of inertia is

$$I = \frac{8}{15}\rho_s \pi R^5 \ .$$

Also, V, the orbital velocity, can be expressed as

$$V = R_o \omega_o$$

where R_o is the radius of the orbit measure from the center of the earth. By using these substitutions, we obtain

$$\Delta\theta_s = n\alpha_d \frac{15\pi}{64}\frac{\rho}{\rho_s}\frac{R_o}{R} \sin 2\theta_s \ . \tag{19}$$

The final expression is seen to be a function of various parameters but only two, ρ and θ_s, are of any importance in reducing or increasing $\Delta\theta_s$. If $87^o < \theta_s < 93^o$ (the spin axis between $\pm$ 3^o of the orbital plane), $\Delta\theta_s$ is reduced by an order of magnitude of its maximum value at $\theta_s = \pi/4$. The effect of atmospheric density is best illustrated by choosing, as a basis for calculation, a solid glass sphere of one foot diameter. Therefore,

$$\rho_s = 2.2 \times 10^3 \ kg/m^3$$

$$R = 1/2 \ ft = 1.5 \times 10^{-1} m \ .$$

The atmospheric density is taken as the maximum occurring during an average sunspot cycle and is shown plotted in Fig. 4.[11] With these values a plot of $\Delta\theta_s/\alpha d \sin 2\theta_s$ versus orbital altitude is shown in Fig. 5. The plot shows that the aerodynamic effect can be reduced to less than one sec of arc per year by orbiting the satellite at 600 miles and requiring the spin axis to be within 3^o of the orbital plane.

The magnitude of the aerodynamic effect is directly proportional to the accommodation coefficient of the satellite surface. This parameter can vary between zero and one and its value is dependent upon the surface properties. Most of the experimental data available predicts the accommodation coefficient to be almost one. However, no experimenters have been able to reproduce the orbital environment which requires a very high speed gas flow combined with an almost perfect vacuum. In view of the results obtained shown in Fig. 5, the possibility is presented of using a lower orbit to magnify the aerodynamic effect which would, in turn, provide an accurate measurement of the accommodation coefficient under the actual orbital conditions which are so difficult to achieve in the laboratory. A complete discussion of this possibility and the results of this analysis is presented in the discussion of results section.

To complete this section on the basic aerodynamic torque, the slowdown of the satellite spin rate will be calculated. The slowdown torque, L_{z_s}, is along the z_s axis and the expression for the slowdown rate

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[11] Orbital Flight Handbook, NASA Sp 33, Part 1, 1963, p. II 34.

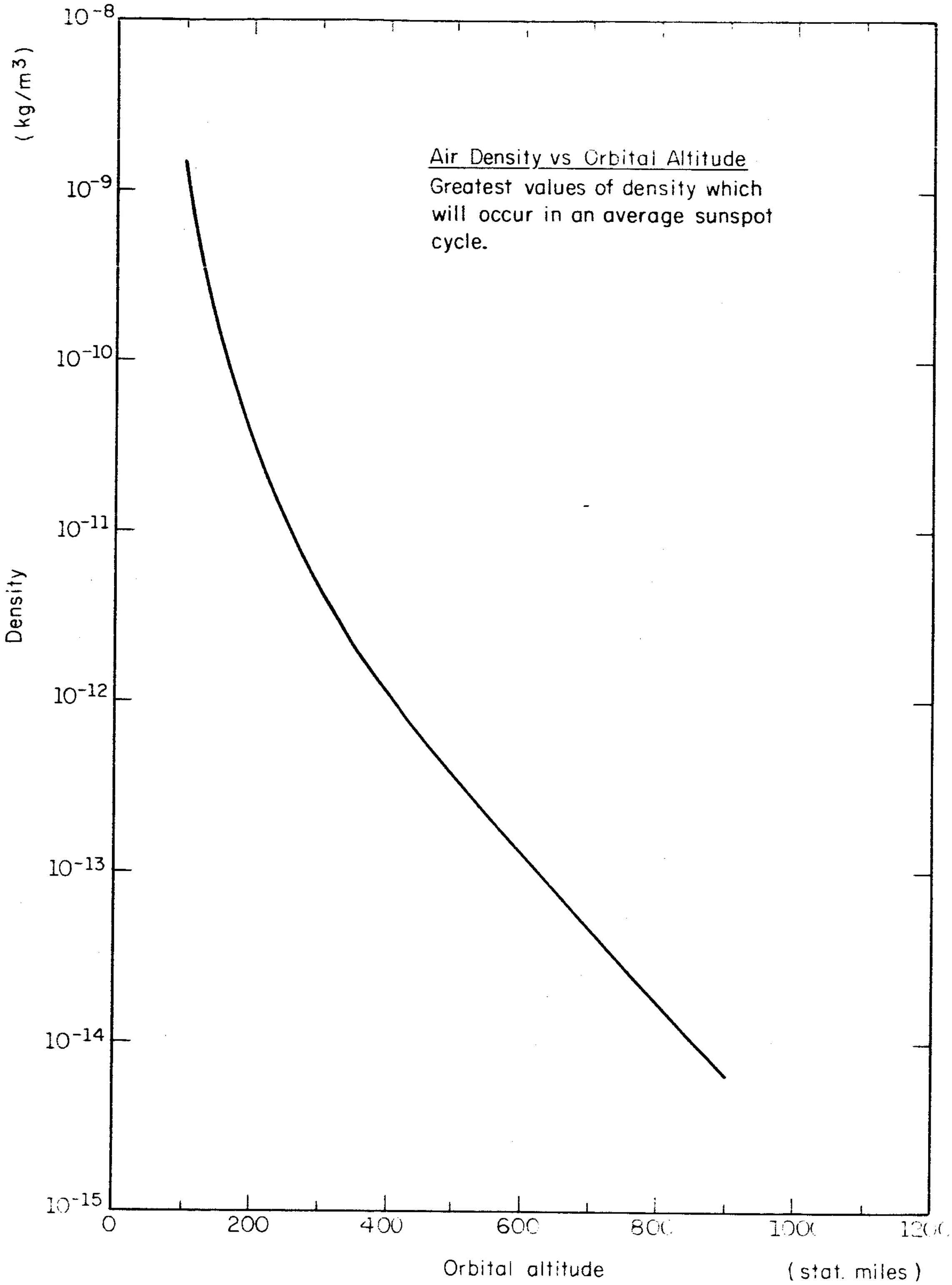

Figure 4. Air density vs. orbital altitude.

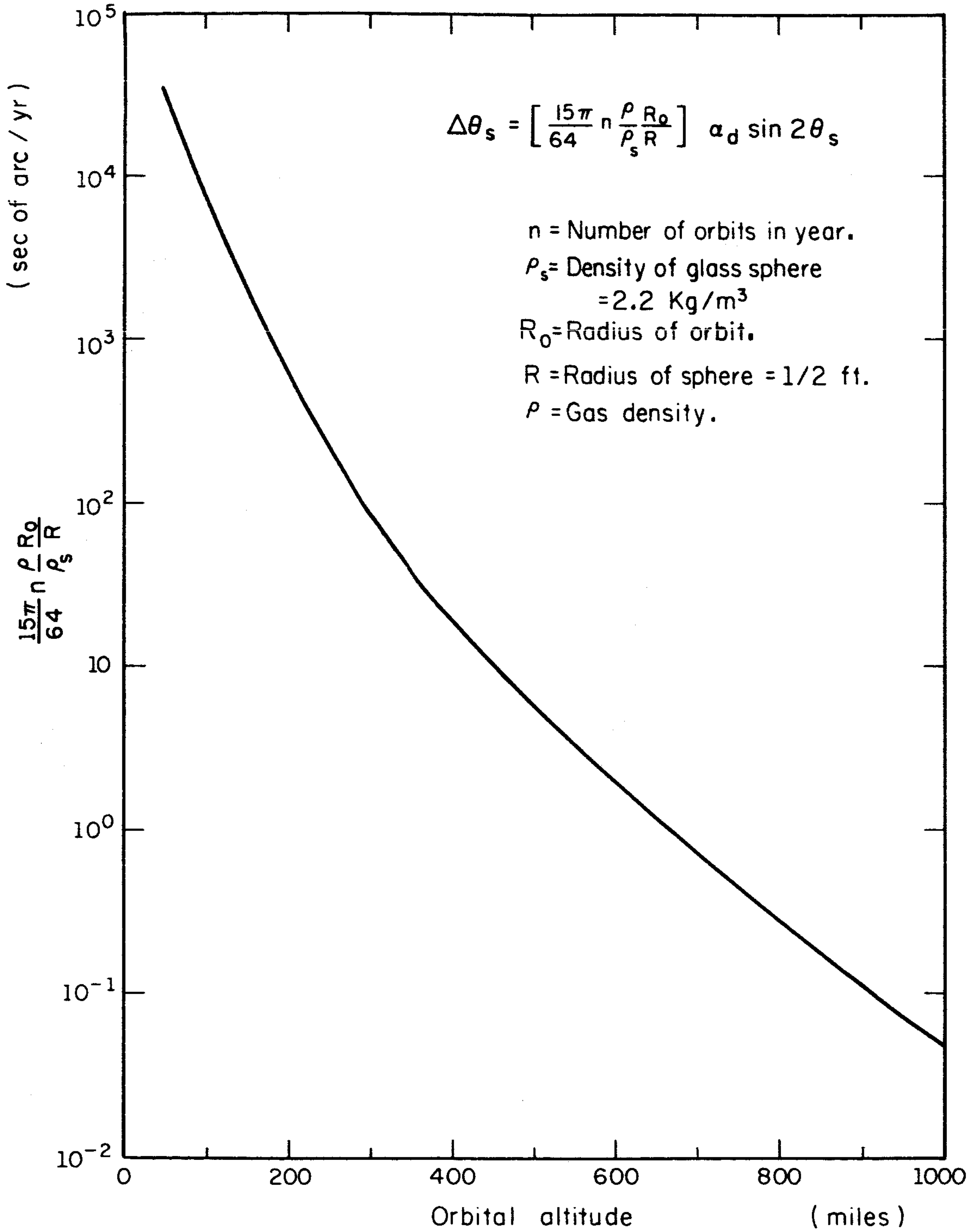

Figure 5. Coefficient of precession rate vs. altitude.

is just

$$\dot{\Omega} = \frac{L_{z_s}}{I} \tag{20}$$

Therefore,

$$\Delta\Omega = \int_0^t \frac{L_{z_s}}{I}\, dt = \frac{n}{I\,\omega_o} \int_0^{2\pi} L_{z_s}\, d\alpha \tag{21}$$

Substitution of L_{z_s} from equation (8) and integrating

$$\Delta\Omega = -\frac{n\pi\, L_o}{I\,\omega_o}\,(5 + \cos^2\theta_s)$$

and using the substitutions for L_o, V, and I this becomes

$$\frac{\Delta\Omega}{\Omega} = -\, n\, \alpha_d\, \frac{15\pi}{64}\, \frac{\rho}{\rho_s}\, \frac{R_o}{R}\, (5 + \cos^2\theta_s)\ . \tag{22}$$

The slowdown rate is seen to be of the order of the precession rate. Substitution of the various parameters at 600 miles yields

$$\frac{\Delta\Omega}{\Omega} \approx 3\times10^{-5} \tag{23}$$

The slowdown effect will therefore be negligible for the altitudes and spin rates to be used in this experiment.

III. CONSIDERATION OF NONUNIFORM SURFACE DISTRIBUTION OF ACCOMMODATION COEFFICIENT

Effects such as roughing of the surface by dust particles, changing of surface properties due to radiation, and nonuniform heating of the surface could cause changes in the accommodation coefficient with respect to position on the surface of the satellite. To study this problem and the resulting motion of the satellite spin axis, we consider the satellite oriented so that the positive spin vector is directed towards the sun. Also assume the spin axis to lie in the plane of the orbit.

Under the above conditions, one-half the surface area of the satellite is in sunlight while the other half is not (see Fig. 6). The surface in the sunlight will then have a higher temperature than that not in sunlight. It may be assumed then that the effect of the nonuniform heating is to cause one-half the surface to have one accommodation coefficient α_d and the other half to have accommodation coefficient α_d''. Since the spin axis is in the plane of the orbit, the satellite presents different parts of its surface to the velocity stream. At some points in its orbit the satellite presents both "hot" and "cold" surfaces to the velocity stream while at other points only a "hot" or a "cold" surface is subjected to molecular impingement. In view of the analysis presented earlier, there is no torque about the spin axis when only a "hot" or a "cold" surface is exposed to the velocity stream (and the spin axis in the plane of the orbit). However, when both "hot" and "cold" surfaces are exposed to the velocity stream a torque will occur and may be calculated in the following way.

Since the spin axis is in the plane of the orbit and pointing towards the sun, the satellite must pass through the shadow of the earth. Assume that while the satellite is in the earth's shadow, the temperature reaches an equilibrium value at all points on its surface. During the period the satellite is in the shadow, no aerodynamic torque acts on the spin axis. Now, referring to Fig. 6, one can visualize a net torque acting on the satellite spin axis every complete orbit. A significant angular displacement could occur after 10^3 such orbits in a year's time.

Unfortunately, it is not convenient to perform the calculations in the same coordinate system as the previous analysis. The coordinate system to be used is shown in Fig. 7. In this system, the spin vector is along the x_s axis and the velocity vector makes an angle α with respect to this vector. The orbit's coordinate system (x, y, z) is defined as before. For calculation purposes it is assumed that the x, y, z axis has the same directions as the x_s, y_s, z_s axis, respectively. Figure 3 illustrates the coordinate system used for surface integration. The vector quantities of interest are:

the satellite spin vector

$$\bar{\Omega} = \Omega i \ , \tag{24}$$

the velocity vector of impinging molecules

$$\bar{V} = -V \cos \alpha \ i - V \sin \alpha \ j \ , \tag{25}$$

the elemental area of satellite surface

$$dA = R^2 \sin \eta \ d\eta \ d\xi \ , \tag{26}$$

the radius vector to the surface of the sphere

$$\bar{R} - R \sin \eta \cos \xi i + R \sin \eta \sin \xi j + R \cos \xi k \ , \tag{27}$$

and the unit normal to the surface

$$\bar{n} = \sin \eta \cos \xi i + \sin \eta \sin \xi j + \cos \eta k \ . \tag{28}$$

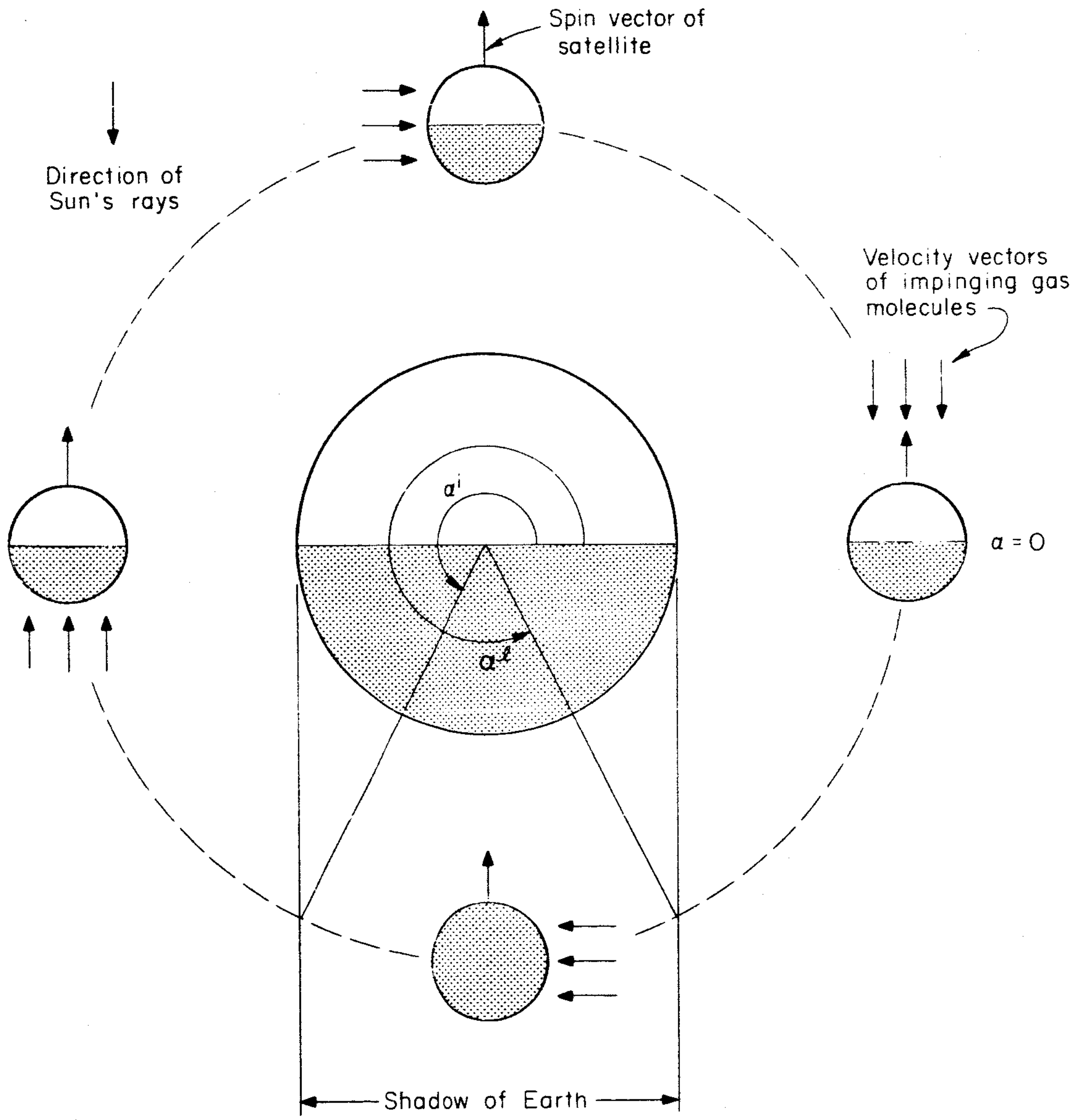

Figure 6. Diagram of nonuniform solar heating effect for satellite in equatorial orbit.

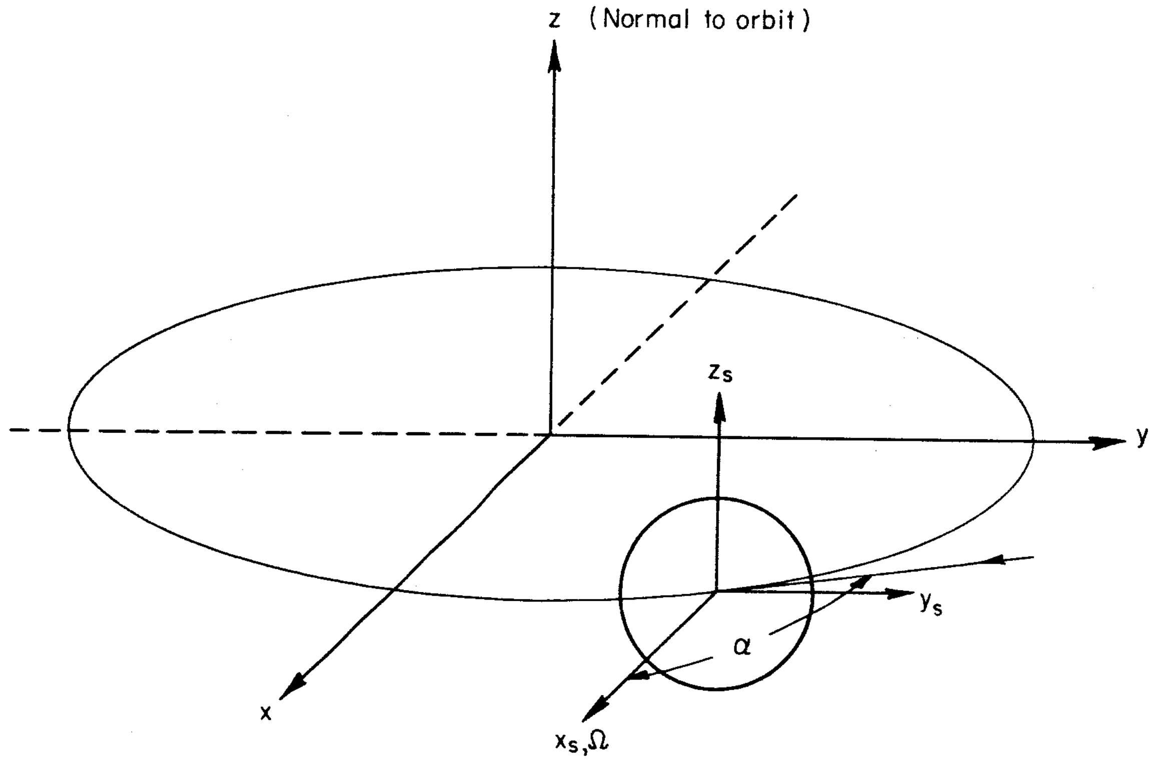

Figure 7. Coordinate system used for nonuniform heating analysis.

As before, the specularly reflected molecules do not enter into the torque analysis.[12] There are now two expressions for elemental torque

$$d\bar{L} = -\alpha_d[\bar{R}\times(\bar{V} - \bar{\Omega}\times\bar{R})(\rho\bar{V}\cdot\bar{n}\ dA)] \tag{29}$$

where $\alpha_d{}'$ must be used for molecules impinging on one-half the surface area and $\alpha_d{}''$ for the other.

Substitution of the vector quantities into the expression for $d\bar{L}$ yields

$$\begin{aligned} d\bar{L} = d\eta\, d\xi\, \alpha_d\, \rho R^2 V \sin^2\eta\{&[RV \cos\eta(\cos\xi \cos\alpha \sin\alpha + \sin\xi \sin^2\alpha) \\ &- R^2\Omega(\cos\xi \cos\alpha + \sin\xi \sin\alpha) \\ &+ R^2\Omega \sin^2\eta(\cos^3\xi \cos\alpha + \cos^2\xi \sin\xi \sin\alpha)]i \\ &+ [-RV \cos\eta(\cos\xi \cos^2\alpha + \sin\xi \sin\alpha \cos\alpha) \\ &+ R^2\Omega \sin^2\eta(\sin\xi \cos^2\xi \cos\alpha + \sin^2\xi \cos\xi \sin\alpha)]j \\ &+ [RV \sin\eta(\sin\xi \cos\xi \cos^2\alpha + \sin^2\xi \sin\alpha \cos\alpha) \\ &- RV \sin\eta(\cos^2\xi \sin\alpha \cos\alpha + \cos\xi \sin\xi \sin^2\alpha) \\ &+ R^2\Omega \sin\eta \cos\eta(\cos^2\xi \cos\alpha + \sin\xi \cos\xi \sin\alpha)]k\}. \end{aligned} \tag{30}$$

To find $\bar{L}$, the integration over η and ξ is performed by first integrated over η from o to π. The integration over η can be performed without considering the difference in accommodation coefficient. The result, yet to be integrated over ξ, becomes,

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[12] The justification for neglecting specularly reflected molecules when α_d changes with position may not be as clear as before when α_d was constant. The resultant force of a specular reflection still passes through the center of mass of the satellite. This force is greater due to impingement on one surface than on the other. Therefore, the net effect is to cause an aerodynamic "lift" to the satellite, but no torque.

$$\bar{L} = \frac{\alpha_d \rho R^2 V\pi}{2} \int_{\xi} d\xi [R^2\Omega(- \cos \xi \cos \alpha - \sin \xi \sin \alpha + \frac{3}{4} \cos^3 \xi \cos \alpha + \frac{3}{4} \sin \xi \cos^2 \xi \sin \alpha) i + \frac{3}{4} R^2\Omega(\sin^2 \xi \cos \xi \sin \alpha + \sin \xi \cos^2 \xi \cos \alpha) j + \frac{4}{3} \frac{RV}{\pi} (\cos 2\xi \sin 2\alpha - \sin 2\xi \cos 2\alpha) k] . \tag{31}$$

To perform the integration over ξ, the difference in accommodation coefficient must be considered. Assume the hot surface, α_d', is the half sphere from $\xi = -\frac{\pi}{2}$ to $\xi = \frac{\pi}{2}$. The cold surface, α_d'', then is from $\xi = \frac{\pi}{2}$ to $\xi = \frac{3\pi}{2}$. The integration must also include only the surface exposed to the velocity stream. The integration is divided into four parts.

$$\bar{L} = \left[\alpha_d' \int_{\alpha - \frac{\pi}{2}}^{\frac{\pi}{2}} d\xi (----) + \alpha_d'' \int_{\frac{\pi}{2}}^{\alpha + \frac{\pi}{2}} d\xi (----) \right]_{o < \alpha < \pi} + \left[\alpha_d'' \int_{\alpha - \frac{\pi}{2}}^{\frac{3\pi}{2}} d\xi (----) + \alpha_d' \int_{\frac{3\pi}{2}}^{\alpha + \frac{\pi}{2}} d\xi (----) \right]_{\pi < \alpha < 2\pi} . \tag{32}$$

The subscripted square bracket serves to indicate that the integration contained in the bracket is valid only within the limits stated on the angle α. When α is beyond these limits, the other bracketed integrations must be used.

Integrating over ξ, the x,y,z components of the vector $\bar{L}$ become:

$$\begin{aligned} L_x &= \rho V R^4 \Omega \frac{\pi}{8} [\alpha_d'(\cos^2 \alpha - 2 \cos \alpha - 3) + \alpha_d''(\cos^2 \alpha + 2 \cos \alpha - 3)] \\ L_y &= \rho V R^4 \Omega \frac{\pi}{8} [\alpha_d'(\sin \alpha + \frac{\sin 2\alpha}{2}) + \alpha_d''(- \sin \alpha + \frac{\sin 2\alpha}{2})] \\ L_z &= \rho V^2 R^3 \frac{1}{3} (\alpha_d' - \alpha_d'')\{[1 - \cos 2\alpha]_{o < \alpha < \pi} + [- 1 + \cos 2\alpha]_{\pi < \alpha < 2\pi} \} . \end{aligned} \tag{33}$$

It is to be noted that only the z component of torque requires the subscripted notation introduced above.

Since the coordinate systems have been set up so that x,y,z have the same direction as x_s, y_s, z_s at the instant in time, the instantaneous torque on the satellite referred to the fixed inertial frame of the orbit has the same components when referred to the coordinates fixed with respect to the satellite. Therefore, at this instant in time

$$L_x = L_{x_s}, \; L_y = L_{y_s}, \text{ and } L_z = L_{z_s} \quad .$$

To describe the motion of the satellite under the action of this torque the reference angles ϕ_s and θ_s are introduced. Let θ_s be the angle between the normal to the orbit, z, and the spin axis, x_s. Initially then, $\theta_s = \frac{\pi}{2}$. Let ϕ_s be the angle between the line of nodes of the orbit, x, and the rising node of the satellite with respect to the orbit plane, y_s. Therefore, $\phi_s = \frac{\pi}{2}$ initially. The motion of the coordinate system attached to the satellite is defined by

$$\begin{aligned} \omega_{x_s} &= -\dot{\phi} \cos\theta \\ \omega_{y_s} &= \dot{\theta} \\ \omega_{z_s} &= \dot{\phi} \sin\theta \; . \end{aligned} \tag{34}$$

Euler's dynamical equations for this system are

$$\begin{aligned} L_{x_s} &= I_{x_s} (\dot{\omega}_{x_s} + \dot{\Omega}) \\ L_{y_s} &= I \dot{\omega}_{y_s} + (I_{x_s} - I) \omega_{z_s} \omega_{x_s} + I_{x_s} \Omega \omega_{z_s} \\ L_{z_s} &= I \dot{\omega}_{z_s} - (I_{x_s} - I) \omega_{x_s} \omega_{y_s} - I_{x_s} \Omega \omega_{y_s} \end{aligned} \tag{35}$$

where $I_{y_s} = I_{z_s} = I$ the moment of inertia of the body. Although the moment of inertia about the spin axis must be larger than the other moments for spin stabilization of the CSL satellite, the difference is considered small enough to neglect in this analysis. Therefore, let

$$I_{x_s} \approx I = \text{the moment of inertia of a sphere.}$$

Making the usual assumption that the spin rate is much larger than $\bar{\omega}_s$, the Euler equations simplify to

$$
\begin{aligned}
L_{x_s} &= I\,\dot{\Omega} \\
L_{y_s} &= I\,\Omega\,\omega_{z_s} = I\,\Omega\,\dot{\phi}\,\sin\theta_s = I\,\Omega\,\dot{\phi} \\
L_{z_s} &= -I\,\Omega\,\omega_{y_s} = -I\,\Omega\,\dot{\theta}_s \quad .
\end{aligned}
\tag{36}
$$

The angular precession can now be found from

$$\Delta\phi_s = \int_o^t \frac{L_{y_s}}{I\Omega}\,dt = \int_o^\alpha \frac{L_{y_s}}{I\Omega}\frac{d\alpha}{\omega_o} \tag{37}$$

and

$$\Delta\theta_s = -\int_o^t \frac{L_{z_s}}{I\Omega} = -\int_o^\alpha \frac{L_{z_s}}{I\Omega}\frac{d\alpha}{\omega_o} \tag{38}$$

where ω_o is the orbital angular velocity and is a constant for a circular orbit. Performing the indicated integration on equations (37) and (38) after substitution of the expressions for L_{y_s} and L_{z_s} given in equation (36) yields

$$\Delta\phi_s = -\frac{\rho V R^4}{I\,\omega_o}\,\frac{\pi}{8}\left\{(\alpha_d' - \alpha_d'')\cos\alpha + (\alpha_d' + \alpha_d'')\,\frac{1}{4}\cos 2\alpha\right\}\Big|_o^\alpha \tag{39}$$

$$\Delta\theta_s = -\frac{\rho V^2 R^3}{3 I\Omega\,\omega_o}\,(\alpha_d' - \alpha_d'')\left\{\pi + \left(-\alpha + \frac{\sin 2\alpha}{2}\right)\Big|_\pi^\alpha\right\} \quad . \tag{40}$$

The limits on α must now be specified in accordance with the assumption that no torque acts on the satellite when it is in the shadow of the earth. The angle, α^i, at which the satellite enters the earth's shadow is (see Fig. 6)

$$\alpha^{i} = \pi + \cos^{-1}\frac{R_e}{R_o} \tag{41}$$

where R_e is the radius of the earth and R_o is the radius of the orbit measured from the center of the earth. The angle, α^{ℓ}, at which the satellite leaves the earth's shadow is

$$\alpha^{\ell} = 2\pi - \cos^{-1}\frac{R_e}{R_o} \quad . \tag{42}$$

The integration between the limits α^{i} and α^{ℓ} contribute nothing to $\Delta\theta_s$ and $\Delta\phi_s$ because the torque is zero in this region. Therefore, the limits on α become

$$\Delta\phi_s = [----]_{o}^{\alpha^{i}} + [----]_{\alpha^{\ell}}^{2\pi}$$

$$\Delta\theta_s = [----]_{\pi}^{\alpha^{i}} + [----]_{\alpha^{\ell}}^{2\pi} \quad .$$

For one complete orbit then

$$\Delta\phi_s = \frac{\pi}{4}\,\frac{\rho V R^4}{I\,\omega_o}\,\frac{R_e}{R_o}\,(\alpha_d{}' - \alpha_d{}'') \tag{43}$$

$$\Delta\theta_s = -\frac{1}{3}\,\frac{\rho V^2 R^3}{I\Omega\,\omega_o}\,(\alpha'_d - \alpha''_d)(\pi - 2\beta_s + \sin 2\beta_s) \tag{44}$$

where

$$\beta_s = \cos^{-1}\frac{R_e}{R_o} \quad .$$

Substituting into these equations the expressions for the moment of inertia of a sphere, $I = \frac{8}{15}\rho_s\,\pi R^5$, and the velocity of the satellite $V = R_o\omega_o$, we obtain

$$\frac{\Delta\phi_s}{\alpha'_d - \alpha''_d} = \frac{15}{32}\,\frac{\rho}{\rho_s}\,\frac{R_e}{R} \tag{45}$$

and

$$\frac{\Delta\theta_s}{\alpha'_d - \alpha''_d} = \frac{15}{24}\,\frac{\rho}{\rho_s}\,\frac{\omega_o}{\Omega}\,\frac{R_o}{R}^{2}\;\frac{\pi - 2\beta_s + \sin 2\beta_s}{\pi} \tag{46}$$

where

$$\beta_s = \cos^{-1} \frac{R_e}{R_o} .$$

Since the orientation of the satellite spin axis with respect to the sun will change due to the motion of the earth about the sun, equations (52) and (53) give the maximum angular displacement of the spin axis that can occur in one orbit. Now, one-quarter of a year after the spin axis was pointing at the sun, the position of the earth in its orbit will cause the satellite spin axis to be normal to the sun's rays. The accommodation coefficient will then be of constant value at all points on the surface because the satellite will have reached an equilibrium temperature at all points on the surface. Then $\Delta\theta_s$ and $\Delta\phi_s$ will be zero for one orbit since $(\alpha'_d - \alpha''_d) = 0$. To obtain the actual variation of $\Delta\theta_s$ and $\Delta\phi_s$ throughout the year would require knowledge of the variation of temperature distribution and accommodation coefficient which is not available. This variation is approximated by assuming $\Delta\theta_s$ and $\Delta\phi_s$ to decrease or increase linearly from its maximum value to zero in one-quarter year. From the plot of the known point shown in Fig. 4 one can easily determine that the total integrated angular displacement for one year would be zero. However, the maximum deflection from an initial value of zero would be large and measurable. This value is just the integrated angular deflection for the half year when $\Delta\theta_s$ and $\Delta\phi_s$ start from zero values, pass through the maximum and back to zero. This integral is approximated as the area of an equilateral triangle with base of 1/2 year and having vertices at the known points seen in Fig. 8

$$(\phi_s \text{ or } \theta_s)(\text{maximum}) = \left(\frac{\text{number of orbits in 1/2 year}}{2}\right)\left(\frac{\Delta\theta_s \text{ or } \Delta\phi_s}{\text{one orbit}}\right) .$$

A plot of the maximum total angular precession is given in Fig. 9 where for purposes of calculation the spin ratio, Ω, material density, ρ_s, and

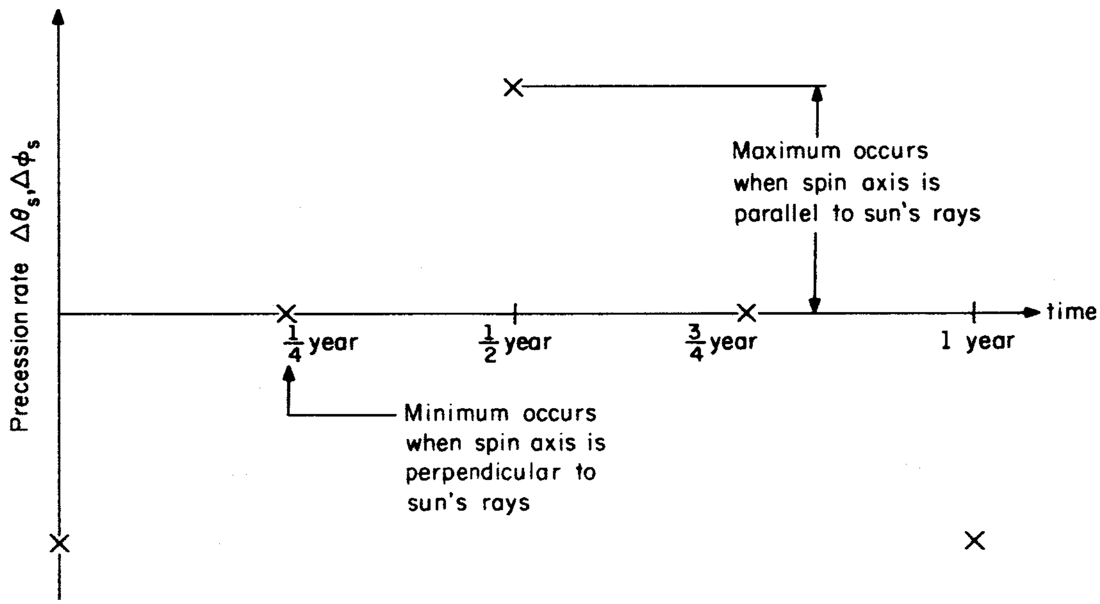

Figure 8. Diagram of known precession rate values during one year due to nonuniform heating.

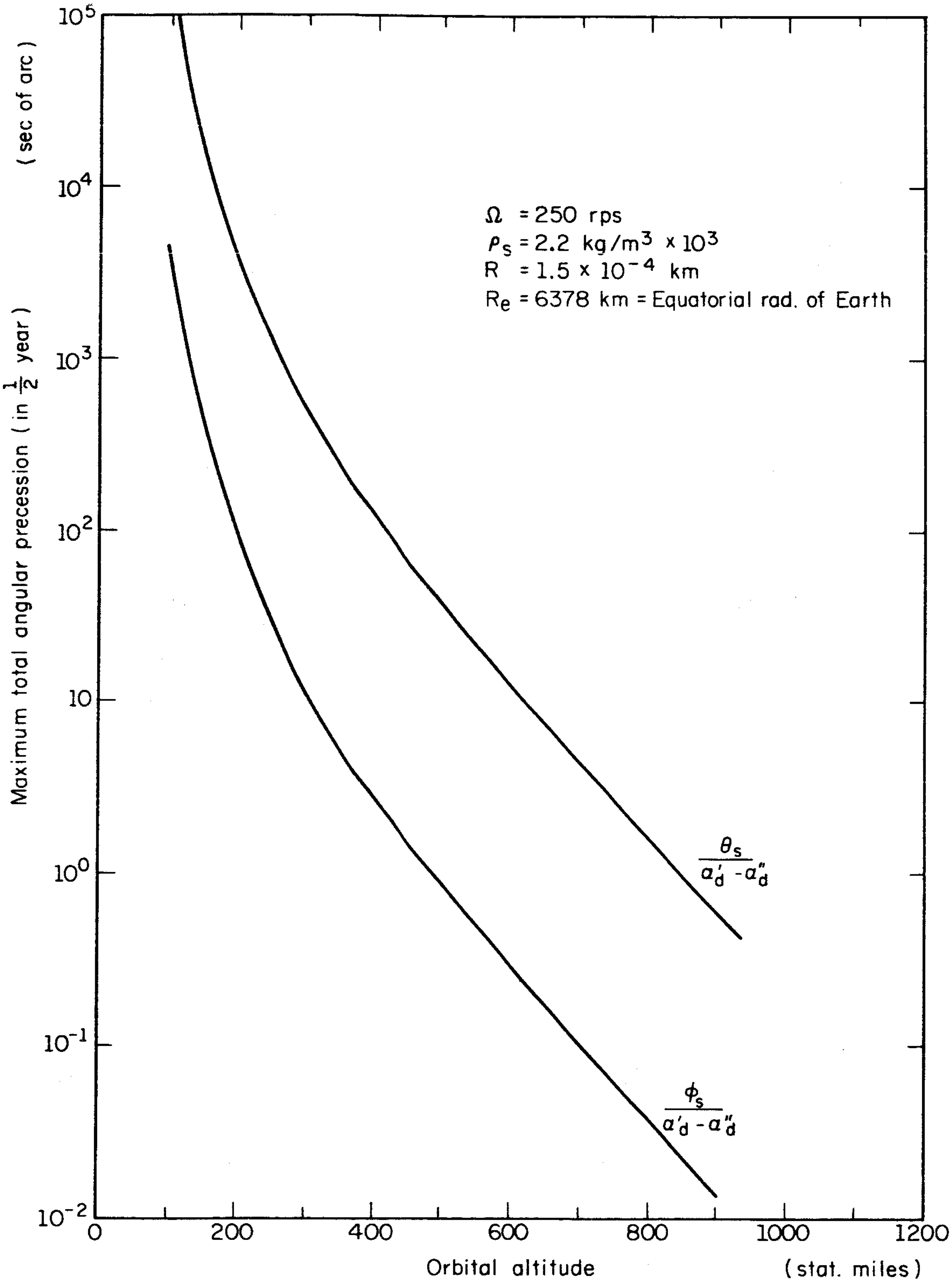

Figure 9. Aerodynamic precession due to nonuniform solar heating vs. altitude.

radius, R, of the gyro where taken to be 250 rev/sec, 2.2×10^3 kg/m^3 and 1.5×10^{-4}km, respectively.

From Fig. 9 it is seen that the nonuniform solar heating effect could be significant at even 600 miles if the accommodation coefficient difference is large due to the temperature difference. Fortunately, experimental data predicts an extremely small change in accommodation coefficient with temperature, if any at all. Very little work has been done in this area and valid experimental evidence of the dependence of accommodation coefficient on surface temperature is not readily available. An experiment by J. K. Roberts (1932) shows an increasing thermal accommodation coefficient with temperature of

$$\frac{\Delta\alpha}{\Delta T_w} \approx \frac{.01}{50^oC}$$

where T_w is the temperature of the surface. Some of the more reliable later experiments on thermal accommodation coefficient indicate a much smaller change of[13]

$$\frac{\Delta\alpha}{\Delta T_w} \approx \frac{.001}{50^oC} \quad .$$

One study of normal momentum transfer by R. E. Stickney and F. C. Hurlbut show no change of accommodation coefficient with temperature;[14] however, their results with respect to temperature effect are not accurate enough to reveal a $\Delta\alpha/\Delta T_w$ of less than $.01/50^oC$.

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[13] Wachman, op. cit., p.8.

[14] R. E. Stickney and F. C. Hurlbut, "Studies of Normal Momentum Transfer by Molecular Beam Techniques," Rarefied Gas Dynamics, ed. J. A. Laurman, Vol. I, 1963, pp. 454-469.

If we take into account the experimental work that has been done on the temperature dependence of accommodation coefficient, it appears that the nonuniform heating effect at 600 miles altitude will not be detrimental to the relativity experiment. However, it is an interesting prospect, in view of the scarcity of experimental data, to think of being able to measure this temperature dependence by orbiting at a lower altitude to magnify the aerodynamic effect. The feasibility of performing this type of experiment will be discussed in the last section of this thesis.

Now consider what the motion of the spin axis would be if the difference in accommodation coefficient were due to a distribution of surface roughness or other surface properties. Analogous to nonuniform heating analysis, consider one-half the spherical surface to have a surface property such that its accommodation coefficient is α'_d and then the other half sphere to have a different surface property such that its accommodation coefficient is α''_d. If we assume the spin axis to be in the plane of the orbit, the equations of motion of the spin axis are the same as those for the nonuniform heating case with one notable exception. In the nonuniform heating case the accommodation coefficient difference went to zero when the satellite was in the earth's shadow. In the nonuniform surface property case the accommodation coefficient difference never goes to zero. Therefore, when the integration over one orbit is evaluated, the expression for torque integrates to zero and no finite integrated motion could be observed for one orbit. The torque is also zero if one considers the spin axis to be in the plane which separates the two half spheres having different accommodation coefficients.

From the discussion and calculated results for the nonuniform surface property effect, one can conclude that the torque arising from a localized accommodation difference (such as a small area damaged by a meteoroid) would be much less than that resulting from the nonuniform heating case, and, therefore, localized accommodation coefficient differences would be negligible in comparison to the torques resulting from other effects. This conclusion is also substantiated by the work of R. D. Palamara, who studied the effect of localized accommodation coefficient differences and arrived at the result that the maximum periodic precession would be less than 4.6×10^{-4} sec of arc per unit radian from variation in α_d on a surface element dA.[15]

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[15] R. D. Palamara, op. cit, p. 68

IV. EFFECT OF REGRESSION OF ORBIT ON SATELLITE MOTION

In the calculation completed in the previous sections, the satellite orbit has been assumed fixed in inertial space. In general, earth orbits are regressing orbits because the earth itself is not a perfect sphere. The principal effect of orbital regression is to change the orientation of the satellite with respect to the orbital plane. Since the aerodynamic torque depends upon this orientation, orbital regression will effect the motion of spin axis of the satellite as seen by an earth based observer. Also, if it is desirable to maintain a specific orientation of the satellite with respect to the orbital plane (say to maximize or minimize a torque), an orbital regression analysis must be made to determine the orbital parameters needed. For completeness the effect of regressing orbit is analyzed and sample results obtained to indicate the spin axis motion.

Fortunately, much of the preliminary work necessary for this calculation has been completed by others and many references are available. An unpublished report by James L. Myers of the Coordinated Science Laboratory on gravity gradient precession was used to set up the necessary equations for the calculations.

The coordinate system to be used is shown in Fig. 10. The motions of the spin axis as seen by the earth based observer are referred to an inertial frame with the Z axis along the North Pole and the X-Y plane being the equatorial plane of the earth with the X axis along the line of vernal equinox.

The transformation of the aerodynamic torque expression of equation (8) to the X, Y, Z coordinate system can be completed in two

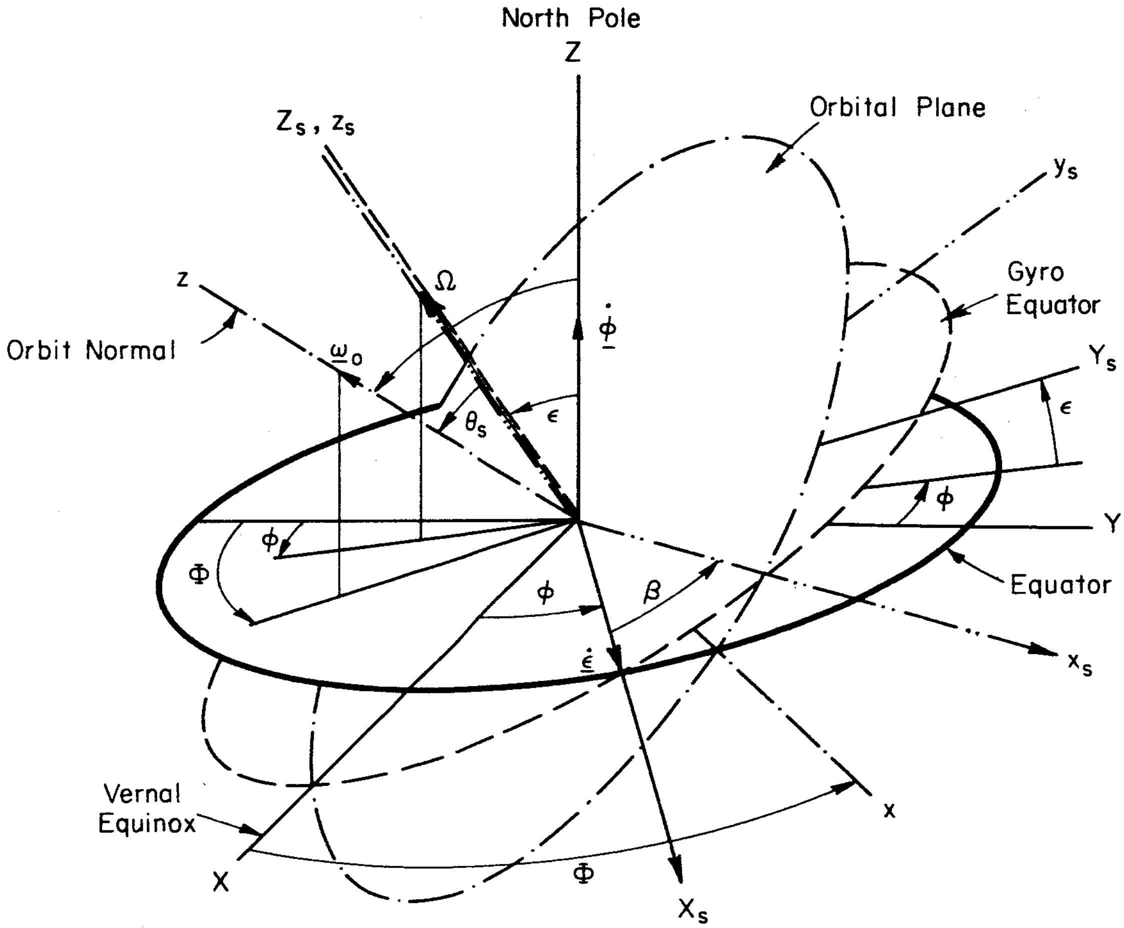

Figure 10. Coordinate systems used for the orbital regression analysis

steps. First let $\phi_s = 0$, then equation (8) becomes

$$L_{x_s} = - L_o \sin \theta_s \frac{\sin 2\alpha}{2} \tag{47}$$

$$L_{y_s} = - L_o \sin \theta_s \cos \theta_s \sin^2\alpha \tag{48}$$

This refers the torque to the orbital plane so that the transformation to the inertial frame is simplified to a transformation of the angle β shown in Fig. 10.

$$L_{X_s} = L_{x_s} \cos \beta - L_{y_s} \sin \beta$$

$$L_{Y_s} = L_{x_s} \sin \beta + L_{y_s} \cos \beta$$

which becomes, using equations (47) and (48),

$$L_{X_s} = - L_o(\sin \theta_s \cos \beta \frac{\sin 2\alpha}{2} - \sin \theta_s \cos \theta_s \sin \beta \sin^2\alpha) \tag{49}$$

$$L_{Y_s} = - L_o(\sin \theta_s \sin \beta \frac{\sin 2\alpha}{2} + \sin \theta_s \cos \theta_s \cos \beta \sin^2\alpha) \ . \tag{50}$$

The orbital regression is taken into account by expressing the angles θ_s and β in terms of the time varying orbital parameters i and Φ, and the spin axis direction angles ϵ and ϕ

$$\begin{aligned} \cos \theta_s &= \sin i \sin \epsilon \cos(\Phi - \phi) + \cos i \cos \epsilon \\ \sin \theta_s \sin \beta &= \sin i \sin(\Phi - \phi) \\ \sin \theta_s \cos \beta &= \sin i \cos \epsilon \cos (\Phi - \phi) - \cos i \sin \epsilon \ . \end{aligned} \tag{51}$$

Now, using the reduced Euler equation for gyros with high spin rates

$$\dot{\phi} = \frac{L_{X_s}}{I\,\Omega \sin \epsilon} \tag{52}$$

$$\dot{\epsilon} = - \frac{L_{Y_s}}{I\,\Omega} \tag{53}$$

Therefore, substituting equations (51) into equations (49) and (50) and substituting this result into the Euler equations, (52) and (53), the motion of the spin axis is given by

$$\dot{\phi} = -\frac{L_o}{I\,\Omega}[\sin i \cot \epsilon \cos(\Phi - \phi) - \cos i]\,\frac{\sin 2\alpha}{2}$$
$$+ \frac{L_o}{I\,\Omega}[\sin i \sin(\Phi - \phi)][\sin i \cos(\Phi - \phi) + \cos i \cot \epsilon]\,\sin^2\alpha \qquad (54)$$

$$\dot{\epsilon} = \frac{L_o}{I\,\Omega}[\sin i \sin(\Phi - \phi)]\,\frac{\sin 2\alpha}{2}$$
$$+ \frac{L_o}{I\,\Omega}[\sin i \cos \epsilon \cos(\Phi - \phi) - \cos i \sin \epsilon][\sin i \cos(\Phi - \phi) \qquad (55)$$
$$+ \cos i \cot \epsilon]\,\sin^2\alpha\ .$$

To find the angular displacement of the spin axis for a given period of time, equations (54) and (55) must be integrated over time. Although i, ϵ, Φ, ϕ, and α are all functions of time, important simplifications can be made by noting the degree of dependence on time of each angle. The angles i, ϵ, and ϕ change by so small an amount that they can be considered constant compared to Φ and α. Now, $\alpha = \omega_o t$, where ω_o is the angular velocity of the satellite about the earth. For the types of orbits being considered, ω_o is large compared to the regression angular velocity $\dot{\Phi} = \Phi/t$. For the period of time of one orbit, Φ can be assumed constant compared to α and an average $\dot{\phi}$ and $\dot{\epsilon}$ can be calculated by integrating equations (54) and (55) over the angle α from 0 to 2π. Therefore,

$$\tilde{\dot{\phi}} = \frac{\omega_o}{2\pi}\int_0^{2\pi} \dot{\phi}\,\frac{d\alpha}{\omega_o} \qquad (56)$$
$$= \frac{1}{2}\,\frac{L_o}{I\,\Omega}\,[\sin i \sin(\Phi - \phi)][\sin i \cos(\Phi - \phi) + \cos i \cot \epsilon]$$

$$\tilde{\epsilon} = \frac{\omega_o}{2\pi} \int_o^{2\pi} \dot{\epsilon} \frac{d\alpha}{\omega_o}$$

$$= \frac{1}{2} \frac{L_o}{I\,\Omega}[\sin i \cos \epsilon \cos(\Phi - \phi) - \cos i \sin \epsilon][\sin i \cos(\Phi - \phi) + \cos i \cot \epsilon] \qquad (57)$$

With these equations then, the angular displacement of the spin axis can be found after a given time if the initial values of i, ϵ, and ϕ are given. The integration must be taken over an integral number of orbits with Φ now being the only dominant function of time. To a first approximation,[16] Φ is given

$$\Phi = \dot{\Phi} t + \Phi_o$$

where Φ_o is an initial condition and

$$\dot{\Phi} = 3\pi \; J_2 \left(\frac{R_e}{R_o} \right) \cos i \; \frac{\text{rad}}{\text{rev}}$$

for a circular orbit of radius R_o and inclination i, where

$$J_2 = 1.082 \times 10^{-3} \quad .$$

For ecliptic orbits with small eccentricity the expression given above is approximately the same. Also, higher order corrections are small.

The integration of the equations is easily performed, but there are many different results possible, depending upon the initial conditions on ϵ, ϕ, i, and Φ. Numerical methods should be used at this point to give

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[16] Orbital Flight Handbook, op. cit., p. IV 21.

the possible results for various initial conditions. However, much information can be learned by taking a particular simple case which reveals the type of solutions to be expected.

In an experiment to measure either the accommodation coefficient or the general relativity effect, the spin axis is required to remain fixed with respect to the orbit plane. This requirement restricts the orbit to be either a near polar or near equatorial orbit because these are the only non-regressing orbits. To illustrate the type of solutions expected when orbital regression is included, the near polar orbit will be used as an example. Then,

$$\sin i \approx i$$

$$\cos i \approx i$$

also require the spin axis to be nearly in the orbital plane. That is, let

$$\epsilon = \frac{\pi}{2} + \delta_e$$

where δ_e is a small angle. Then

$$\sin \epsilon \approx 1$$

$$\cos \epsilon \approx -\delta_e \quad .$$

With these assumptions, equations (56) and (57) for the average angular motion of the spin axis become,

$$\tilde{\dot{\phi}} = \frac{L_o}{2I\Omega}\left[i^2 \frac{\sin 2(\Phi-\phi)}{2} - \delta_e\, i \sin(\Phi-\phi)\right] \tag{58}$$

$$\tilde{\dot{\epsilon}} = \frac{L_o}{2I\Omega}\left[i^2\delta_e \cos^2(\Phi-\phi) - \delta_e^2 \cos(\Phi-\phi) + i\cos(\Phi-\phi) - \delta_e\right] \tag{59}$$

Since the angles i and δ_e are small, the above equations can be simplified by neglecting terms having coefficients of order higher than i or δ_e. The result is then,

$$\tilde{\dot{\phi}} \approx 0 \qquad \text{or} \qquad \phi = \phi_o = \text{constant} \tag{60}$$

$$\tilde{\dot{\epsilon}} \approx \frac{L_o}{2I\Omega}\left[i\cos(\Phi-\phi) - \delta_e\right] . \tag{61}$$

Now,

$$\cos(\Phi-\phi) = \cos(\dot{\Phi}t + \Phi_o - \phi_o) \quad .$$

Therefore, letting $\Phi_o-\phi_o = 0$ the motion of the spin axis is given by

$$\tilde{\dot{\epsilon}} = \frac{L_o}{2I\Omega}\left[i\cos\dot{\Phi}t - \delta_e\right] \quad . \tag{62}$$

Therefore,

$$\begin{aligned}\Delta\epsilon &= \frac{L_o}{2I\Omega}\int_o^t (i\cos\dot{\Phi}t - \delta_e)dt \\ &= \frac{L_o}{2I\Omega}\int_o^t \left[i\cos\left(\frac{\dot{\Phi}}{\omega_o}\alpha\right) - \delta_e\right]\frac{d\alpha}{\omega_o}\end{aligned} \tag{63}$$

where n is the number of orbits in a time, T. Therefore,

$$\Delta\epsilon = \frac{L_o}{2I\Omega}\left[\frac{i}{\dot{\Phi}}\sin\left(\dot{\Phi}\,\frac{2n\pi}{\omega_o}\right) - \frac{2n\pi}{\omega_o}\,\delta_e\right]$$

or

$$\Delta\epsilon = \frac{L_o\, n\pi}{2I\Omega\, \omega_o}\left[\frac{2}{\dot{\Phi}T}\, i\sin\dot{\Phi}T - 2\delta_e\right]$$

where $T = 2n\pi/\omega_o$. Substituting for L_o and I, this becomes

$$\Delta\epsilon = n\,\alpha_d \frac{15\pi}{64}\frac{\rho}{\rho_s}\frac{R_o}{R}\left[\frac{2}{\dot{\Phi}T}\, i \sin\dot{\Phi}T - 2\delta_e\right] \tag{64}$$

The coefficient of the bracketed term is the same as found for the non-regressing orbit analysis and is plotted in Fig. 5. In fact, the second term

$$n\,\alpha_d \frac{15\pi}{64}\frac{\rho}{\rho_s}\frac{R_o}{R}(-2\delta_e)$$

is exactly the result one would expect if the orbit was non-regressing and $\theta_s = \frac{\pi}{2} + \delta_e$. Therefore, the effect of regression is seen to add an oscillating term having the orbital regression frequency and magnitude dependent upon the orbital inclination.

At 600 miles for a solid glass satellite of one-foot diameter and time equal to one year

$$n\,\alpha_d \frac{15\pi}{64}\frac{\rho}{\rho_s}\frac{R_o}{R} \simeq 2\,\alpha_d \quad \text{sec arc}$$

$$\dot{\Phi} = 35\,\frac{\text{rad}}{\text{year}}$$

$$T = 1 \text{ year} \quad .$$

Then, letting $\sin\dot{\Phi}T$ be maximum (± 1), we have

$$\frac{\Delta\epsilon}{\alpha_d} \simeq 2[-2\,\delta_e \pm .056\, i] \text{ sec/arc} \quad . \tag{65}$$

This result shows that the effect of orbital regression for the condition used is small if δ_e is the same order as i. However, if i is allowed to be an order of magnitude greater than δ_e, the orbital regression effects could be of the same order as the aerodynamic effect.

The most important conclusion drawn from the calculations of this section is that when orbital regression is included in the analysis, the results will be separable into two parts. One term will be of the same type as for the non-regressing orbit. The other part of the solution will contain terms of the orbital regression frequency. These last terms cannot in general be neglected because they will depend upon the orbital parameters and be of various magnitudes. Also, if it is desired to keep the spin axis fixed with respect to the orbital plane, the regression results will give the accuracy required on i, ϵ, and ϕ to retain a certain initial orientation.

V. DISCUSSION OF RESULTS

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This analysis has consisted of first obtaining the aerodynamic torque equations for a spinning spherical satellite and finding the motion of the spin axis due to this torque. The torque was found to depend upon the accommodation coefficient of the satellite surface and the orientation of the satellite with respect to the orbital plane. Consideration was therefore given to the effects of nonuniform accommodation coefficient distribution and time dependent satellite orientation. The former was analyzed by considering nonuniform heating of the satellite surface which was assumed to cause a nonuniform distribution of accommodation coefficient. The effect of time dependent orientation of the satellite was analyzed by including orbital regression effects which cause a change in the orientation of the satellite with respect to the orbital plane. The results of all the calculations were applied to the satellite proposed by the Coordinated Science Laboratory to measure a general relativity effect. These calculated results will now be discussed; first with respect to reducing the aerodynamic effect so as not to obstruct the measurement of the relativity effect and, second, with respect to the feasibility of utilizing the aerodynamic effect to measure accommodation coefficient. AUTHOR

Minimizing the Aerodynamic Torque

The experiment proposed by the Coordinated Science Laboratory to measure a general relativity effect consists of measuring the angular displacement of the satellite spin axis from its original position after a one-year period of time. The predicted angular displacement due to the relativity effect alone is 5 to 7 sec of arc per year. It is desirable in regard to the relativity experiment to reduce the aerodynamic effect to a value of at least less than one-tenth the predicted relativity effect. From the results calculated in this report, the reduction of the aerodynamic effect is possible providing proper satellite orientation is maintained. Figure 5 shows that if the satellite is orbited at an altitude of 600 miles or more and the spin axis is restricted to lie within 3^{o} of the orbital plane, then an angular displacement of less than .2 sec of arc per year is to be expected. These are reasonable requirements since the experiment requires the spin axis to be nearly in the plane of the orbit to produce the relativity effect. The 600 mile altitude limitation is approximately the altitude originally proposed for the experiment and has since served as one of experimental parameters.

The nonuniform heating analysis was considered to be of importance to the relativity experiment because if an equatorial orbit were used there would be very little that could be done to prevent nonuniform heating of the surface of the solid satellite. The results shown in Fig. 9 indicate that nonuniform heating should not be of importance if the available experimental work on accommodation coefficient change with temperature can be relyed upon. Taking the maximum $\Delta\alpha/\Delta T_N$ as $.01/50^{o}$ and assuming a pessimistic value of 50^{o} change in temperature, the precession rate at 600 miles due to nonuniform heating is less than .2 sec of arc per year. Therefore, the use of the equatorial orbit for the relativity experiment is permissible from aerodynamic considerations.

The orbital regression analysis is of importance because from it the orbital parameters are determined which allow the spin axis to remain relatively fixed with respect to the orbital plane. The fixing of the spin axis to the orbital plane for a period of a year is necessary not only to reduce the aerodynamic effect but also to maximize the relativity effect and minimize gravity gradient precession. Therefore, the requirement of near zero orbit inclination for the case of a polar orbit is compatible with the relativity experiment. This compatibility is to be expected for either the polar or equatorial orbits and the various initial conditions possible.

It can then be concluded that the aerodynamic effects can be reduced to less than one-tenth the relativity effect by orbiting at 600 miles or above and requiring the spin axis to be within 3° of the orbital plane.

Feasibility of Utilizing the Aerodynamic Effect to Measure Accommodation Coefficients

The feasibility of utilizing the aerodynamic effect to measure accommodation coefficients can be derived from the results calculated in this report. In order to measure the aerodynamic precession, we should minimize other effects such as the relativity effect and gravity gradient effect. The magnification of the aerodynamic effect can be achieved by using a lower orbit which, due to the higher gas density, causes a substantial increase in aerodynamic torque. However, as the density increases so does the aerodynamic drag and therefore the lifetime of the satellite is greatly reduced when lower orbits are used. To insure a lifetime of at least two years for the type of satellite proposed, an altitude of 300 miles or more must be used. At 300 miles the aerodynamic precession is about 100 α_d sec of arc per year (see Fig. 5) which is well above the relativity effect. To achieve this large amount of precession the spin axis must be oriented so as to be 45^o out of the orbital plane [see Eq. (19)]. This also happens to be the orientation that makes gravity gradient precession maximum. However, gravity gradient precession is known to a high degree of accuracy and can, if necessary, be subtracted from the data. The orbital regression effects can also be taken into account through Eqs. (56) and (57), and these are now more complicated than the result given in Eq. (64). In fact, all the torque analysis completed by the Coordinated Science Laboratory in regard to the relativity experiment are also applicable to an accommodation coefficient experiment. All these other precession effects cause, at the most, .1 sec of arc per year precession and this magnitude is not dependent upon altitude to any appreciable extent.

It appears then that the 100 α_d sec of arc per year precession can be measured to an accuracy of one part in one hundred since all precessions greater than .1 sec of arc per year can be compensated for. This accuracy is directly dependent upon the accurate determination of the atmospheric density, ρ, which appears in Eqs. (19), (54), and (55). The fundamental problem associated with obtaining an accurate atmospheric density to use in the equations is that the density is continually changing not only seasonably but also during each orbit as the satellite passes from the dark into the sunlight side of the earth. In the calculations of this thesis the density has conveniently been assumed to be constant. The exact results can be obtained, however, by using the density as a function of time in the original equations. The variation in density can be determined to the desired accuracy by using the decrease in orbital angular velocity of the satellite itself to calculate the density. This is the common procedure used to determine atmospheric density and is believed accurate to at least one part in one hundred.

It can then be concluded that the accommodation coefficient of the satellite surface can be measured to an accuracy of one part in one hundred by the method outlined above. The exact final analysis which would include orbital regression and time dependent atmospheric density effects would of course require evaluation by a numerical method. The results presented in this thesis are, however, an indication of what is to be expected quantitatively in measuring accommodation coefficients by the satellite experiment proposed.

The results shown in Fig. 9 indicate that it might also be possible to measure the temperature dependence of the accommodation coefficient at an orbit of 300 miles. At this altitude, the angular displacement of the spin axis due to nonuniform heating is about $572(\alpha_d' - \alpha_d'')$ sec of arc per year. Although the accommodation coefficient is thought be be almost unity, the difference $(\alpha_d' - \alpha_d'')$ is believed to be small. If $\alpha_d' - \alpha_d''$ were as large as .01, the angular displacement would then be only 5.72 sec of arc per year which is about the limit of accuracy with which precession can be measured. With the experience the Coordinated Science Laboratory will gain from measurement of the relativity precession, the accuracy of measurements may be increased by an order of magnitude. This would enable us to measure accommodation coefficient differences to less than .001. Since gravity gradient precession is zero for satellite orientation needed to measure temperature effects, only the relativity precession would have to be compensated for in this type experiment.

It can then be concluded that the temperature dependence of accommodation coefficient can be measured only if an order of magnitude increase in accuracy of precession measurements is realized.

VI. CONCLUSIONS

The aerodynamic torque on a spinning spherical satellite has been analyzed and the feasibility has been studied of either reducing the torque or, on the other hand, utilizing it to measure accommodation coefficients of satellite surfaces. From the results of the analysis, two conclusions were obtained. First, the precession resulting from the aerodynamic torque can be reduced to less than .2 sec of arc per year by orbiting at 600 miles or above and requiring the spin axis to be within 3° of the orbital plane. Second, accommodation coefficients of satellite surfaces can be measured by orbiting at 300 miles with the spin axis 45° out of the plane of the orbit. The nonuniform heating analysis shows that the effect is negligible in regard to the relativity experiment and possible measurement of the temperature dependence of accommodation coefficients requires greater accuracy in precession measurements than now proposed. It was also found that orbital regression effects can be reduced by suitable choice of the initial orbital parameters.

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13. ABSTRACT

A spinning spherical satellite has been proposed by the Coordinated Science Laboratory to measure a general relativity effect which is predicted to cause the spin axis of the satellite to precess 5 to 7 sec of arc per year. Measurement of the precession is realized by a unique method or read-out which utilizes terrestrial sightings of sunlight reflected from mirrors on the satellite's surface. The measurement of such a small precession rate required an investigation of other torques producing effects which could possibly prevent the isolation of the relativity effect. In this respect, the analysis to determine the aerodynamic torque was initiated.

The study consists of first obtaining the general analytical expression for aerodynamic torque which is found to depend upon the orientation of the satellite and the accommodation coefficient of the surface. Consideration is given to the effect of nonuniform distribution of accommodation coefficient and orbital regression effects which cause a change in satellite orientation with time. When these results are applied to the CSL satellite, two conclusions are evident. These conclusions are discussed in the report, and the feasibility of performing a satellite experiment to measure the accommodation coefficient by the technique proposed by the Coordinated Science Laboratory is investigated. (Author)

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